## Answers

34. 18
35. 39.6
36. 39.6
37. 19.8
38. 25.5
39. 10.5
40. 25.5
41. 21
42. $41^{\circ}$
43. $49^{\circ}$
44. $82^{\circ}$
45. $98^{\circ}$
46. $\mathrm{m} \angle 1=57^{\circ} ; \mathrm{m} \angle 2=66^{\circ} ; \mathrm{m} \angle 3=$ $33^{\circ} ; \mathrm{m} \angle 4=114^{\circ} ; \mathrm{m} \angle 5=57^{\circ}$
47. $\mathrm{m} \angle 1=37^{\circ} ; \mathrm{m} \angle 2=53^{\circ} ; \mathrm{m} \angle 3=$ $90^{\circ} ; \mathrm{m} \angle 4=37^{\circ} ; \mathrm{m} \angle 5=53^{\circ}$
48. $R T=S U=2 \sqrt{10}$, so $\overline{R T} \cong \overline{S U}$. Slope of $\overline{R T}=-3$, and slope of $\overline{S U}=\frac{1}{3}$, so $\overline{R T} \perp \overline{S U}$. The coordinates of the mdpt. of $\overline{R T}$ and $\overline{S U}$ are $(-4,-3)$, so $\overline{R T}$ and $\overline{S U}$ bisect each other. So the diags. of $R S T U$ are $\cong \perp$ bisectors of each other.
49. $E G=F H=3 \sqrt{2}$, so $\overline{E G} \cong \overline{F H}$. Slope of $\overline{E G}=-1$, and slope of $\overline{F H}=1$, so $\overline{E G} \perp \overline{F H}$. The coordinates of the mdpt. of $\overline{E G}$ and $\overline{F H}$ are $\left(\frac{7}{2},-\frac{1}{2}\right)$, so $\overline{E G}$ and $\overline{F H}$ bisect each other. So the diags. of $E F G H$ are $\cong \perp$ bisectors of each other.
50. Not valid; by Thm. 6-5-2, if the diags. of a $\square$ are $\cong$, then the $\square$ is a rect. By Thm. 6-5-4, if the diags. of a $\square$ are $\perp$, then the $\square$ is a rhombus. If a $\square$ is both a rect. and a rhombus, then the $\square$ is a square. To apply this chain of reasoning, you must first know that $E F R S$ is a $\square$.
51. valid
52. valid

6-4 Properties of Special Parallelograms (pp. 408-415)

## EXAMPLES

In rectangle $J K L M$,
$K M=52.8$, and $J M=45.6$.
Find each length.


■ KL
$J K L M$ is a $\square$.
Rect. $\rightarrow \square$
$K L=J M=45.6$
$\square \rightarrow$ opp. sides $\cong$
■ $N L$

$$
\begin{array}{cc}
J L=K M=52.8 & \text { Rect. } \rightarrow \text { diags. } \cong \\
N L=\frac{1}{2} J L=26.4 & \square \rightarrow \text { diags. bisect } \\
& \text { each other }
\end{array}
$$

- $P Q R S$ is a rhombus.

Find $\mathrm{m} \angle Q P R$, given that
$\mathrm{m} \angle Q T R=(6 y+6)^{\circ}$ and
$\mathrm{m} \angle S P R=3 y^{\circ}$.

$\mathrm{m} \angle Q T R=90^{\circ} \quad$ Rhombus $\rightarrow$ diags. $\perp$
$6 y+6=90 \quad$ Substitute the given value.

$$
y=14 \quad \text { Solve for } y
$$

$\mathrm{m} \angle Q P R=\mathrm{m} \angle S P R \quad$ Rhombus $\rightarrow$ each
$m \angle Q P R=3(14)^{\circ}=42^{\circ} \quad$ diag. bisects opp. $\angle$

- The vertices of square $A B C D$ are $A(5,0)$, $B(2,4), C(-2,1)$, and $D(1,-3)$. Show that the diagonals of square $A B C D$ are congruent perpendicular bisectors of each other.
$A C=B D=5 \sqrt{2}$
slope of $\overline{A C}=-\frac{1}{7}$
Diags. are $\cong$.
slope of $\overline{B D}=7$
Product of slopes is -1 , so diags. are $\perp$.
mdpt. of $\overline{A C}$

$$
=\text { mdpt. of } \overline{B D}=\left(\frac{3}{2}, \frac{1}{2}\right)
$$

Diags. bisect each other.

## EXERCISES

In rectangle $A B C D, C D=18$, and $C E=19.8$. Find each length.
34. $A B$
35. $A C$
36. $B D$
37. $B E$


In rhombus $W X Y Z, W X=7 a+1$,
$W Z=9 a-6$, and $V Z=3 a$.
Find each measure.
38. $W Z$
39. $X V$
40. $X Y$
41. $X Z$


In rhombus RSTV, $\mathrm{m} \angle T Z V=(8 n+18)^{\circ}$, and $\mathrm{m} \angle S R V=(9 n+1)^{\circ}$.
Find each measure.
42. $\mathrm{m} \angle T R S$
43. $\mathrm{m} \angle R S V$
44. $\mathrm{m} \angle S T V$
45. $\mathrm{m} \angle T V R$


Find the measures of the numbered angles in each figure.
46. rectangle $M N P Q$
47. rhombus $C D G H$



Show that the diagonals of the square with the given vertices are congruent perpendicular bisectors of each other.
48. $R(-5,0), S(-1,-2), T(-3,-6)$, and $U(-7,-4)$
49. $E(2,1), F(5,1), G(5,-2)$, and $H(2,-2)$

## 6-5 Conditions for Special Parallelograms (pp. 418-425)

## EXAMPLES

- Determine if the conclusion is valid. If not, tell what additional information is needed to make it valid.
Given: $\overline{L P} \perp \overline{K N}$


Conclusion: $K L N P$ is a rhombus.
The conclusion is not valid.
If the diagonals of a parallelogram are
perpendicular, then the parallelogram is a rhombus. To apply this theorem, you must first know that $K L N P$ is a parallelogram.

## EXERCISES

Determine if the conclusion is valid. If not, tell what additional information is needed to make it valid.

50. Given: $\overline{E R} \perp \overline{F S}, \overline{E R} \cong \overline{F S}$

Conclusion: $E F R S$ is a square.
51. Given: $\frac{\overline{E R}}{\overline{E R}}$ and $\overline{F S}$ bisect each other. $\overline{E R} \cong \overline{F S}$
Conclusion: $E F R S$ is a rectangle.
52. Given: $\overline{E F}\|\overline{R S}, \overline{F R}\| \overline{E S}, \overline{E F} \cong \overline{E S}$ Conclusion: $E F R S$ is a rhombus.

- Use the diagonals to tell whether a parallelogram with vertices $P(-5,3)$, $Q(0,1), R(2,-4)$, and $S(-3,-2)$ is a rectangle, rhombus, or square. Give all the names that apply.

$$
\begin{array}{ll}
P R=\sqrt{98}=7 \sqrt{2} & \text { Distance Formula } \\
Q S=\sqrt{18}=3 \sqrt{2} & \text { Distance Formula }
\end{array}
$$

Since $P R \neq Q S, P Q R S$ is not a rectangle and not a square.

$$
\begin{array}{ll}
\text { slope of } \overline{P R}=\frac{7}{-7}=-1 & \text { Slope Formula } \\
\text { slope of } \overline{Q S}=\frac{3}{3}=1 & \text { Slope Formula }
\end{array}
$$

Since the product of the slopes is -1 ,
the diagonals are perpendicular. $P Q R S$ is a rhombus.

Use the diagonals to tell whether a parallelogram with the given vertices is a rectangle, rhombus, or square. Give all the names that apply.
53. $B(-3,0), F(-2,7), J(5,8), N(4,1)$
54. $D(-4,-3), H(5,6), L(8,3), P(-1,-6)$
55. $Q(-8,-2), T(-6,8), W(4,6), Z(2,-4)$

## 6-6 Properties of Kites and Trapezoids (pp. 427-435)

## EXAMPLES

- In kite $P Q R S, \mathrm{~m} \angle S R T=24^{\circ}$, and $m \angle T S P=53^{\circ}$. Find $m \angle S P T$.
$\triangle P T S$ is a right triangle. $\mathrm{m} \angle S P T+\mathrm{m} \angle T S P=90^{\circ}$

Kite $\rightarrow$ diags. $\perp$ Acute is of rt. $\triangle$ are comp.
$\mathrm{m} \angle S P T+53=90 \quad$ Substitute 53 for $m \angle T S P$. $\mathrm{m} \angle S P T=37^{\circ} \quad$ Subtract 53 from both sides.

Find $\mathrm{m} \angle D$

$$
\begin{aligned}
\mathrm{m} \angle C+\mathrm{m} \angle D & =180^{\circ} \\
51+\mathrm{m} \angle D & =180 \\
\mathrm{~m} \angle D & =129^{\circ}
\end{aligned}
$$



Same-Side Int. \&s Thm. Substitute 51 for $m \angle C$. Subtract.

$J P=32.5$, and $H L=50$.
Find $P N$.
$\overline{J N} \cong \overline{H L}$
Isosc. trap. $\rightarrow$ diags. $\cong$
$J N=H L=50$
Def. of $\cong$ segs
$J P+P N=J N \quad$ Seg. Add. Post.
$32.5+P N=50$
Substitute
$P N=17.5$ Subtract 32.5 from both sides.

Find WZ.

$A B=\frac{1}{2}(X Y+W Z)$
Trap. Midsegment Thm.
$73.5=\frac{1}{2}(42+W Z)$
Substitute.
Multiply both sides by 2.
Solve for WZ.

## EXERCISES

In kite $W X Y Z, \mathrm{~m} \angle V X Y=58^{\circ}$, and $\mathrm{m} \angle Z W X=50^{\circ}$.
Find each measure.
56. $\mathrm{m} \angle X Y Z$
57. $\mathrm{m} \angle Z W V$
58. $\mathrm{m} \angle V Z W$
59. $\mathrm{m} \angle W Z Y$


Find each measure.
60. $\mathrm{m} \angle R$ and $\mathrm{m} \angle S$

61. $B Z$ if $Z H=70$ and $E K=121.6$

62. $M N$

63. $E Q$

64. Find the value of $n$ so that $P Q X Y$ is isosceles.


Give the best name for a quadrilateral whose vertices have the given coordinates.
65. $(-4,5),(-1,8),(5,5),(-1,2)$
66. $(1,4),(5,4),(5,-4),(1,-1)$
67. $(-6,-1),(-4,2),(0,2),(2,-1)$

## Answers

53. rhombus
54. rect.
55. rect., rhombus, square
56. $64^{\circ}$
57. $25^{\circ}$
58. $65^{\circ}$
59. $123^{\circ}$
60. $\mathrm{m} \angle R=126^{\circ} ; \mathrm{m} \angle S=54^{\circ}$
61.51 .6
61. 48.5
62. 3.5
63. $n=3$ or $n=-3$
64. kite
65. trap.
66. isosc. trap.

## Chapter Test

## Organizer

Objective: Assess students' mastery of concepts and skills in Chapter 6.

Online Edition

## Resources

## Assessment Resources

Chapter 6 Tests

- Free Response
(Levels A, B, C)
- Multiple Choice
(Levels A, B, C)
- Performance Assessment


## Teacher One Stop"

Test \& Practice Generator

## StateResonces

Tell whether each figure is a polygon. If it is a polygon, name it by the number of its sides.
1.

2.

3. The base of a fountain is in the shape of a quadrilateral, as shown. Find the measure of each interior angle of the fountain.
4. Find the sum of the interior angle measures of a convex nonagon
5. Find the measure of each exterior angle of a regular 15 -gon. $24^{\circ}$

3.
$\mathrm{m} \angle A=96^{\circ}$;
$\mathrm{m} \angle B=112^{\circ}$;
$\mathrm{m} \angle C=64^{\circ}$;
$\mathrm{m} \angle D=88^{\circ}$
6. In $\square E F G H, E H=28, H Z=9$, and
$F H=18 ;$
7. JKLM is a parallelogram.
$\mathrm{m} \angle E H G=145^{\circ}$. Find $F H$ and $\mathrm{m} \angle F E H . \mathrm{m} \angle F E H \quad$ Find $K L$ and $\mathrm{m} \angle L . K L=17 ; \mathrm{m} \angle L=52^{\circ}$

8. Three vertices of $\square P Q R S$ are $P(-2,-3), R(7,5)$, and $S(6,1)$. Find the coordinates of $Q \cdot(-1,1)$
9. Show that $W X Y Z$ is a parallelogram for $a=4$ and $b=3$.

10. Determine if $C D G H$ must be a parallelogram. Justify your answer.

11. Show that a quadrilateral with vertices $K(-7,-3), L(2,0), S(5,-4)$, and $T(-4,-7)$ is a parallelogram.
12. In rectangle $P L C M$, $L C=19$, and $L M=23$. Find $P T$ and $P M$. $P T=11.5 ; P M=19$

13. In rhombus $E H K N$, $\mathrm{m} \angle N Q K=(7 z+6)^{\circ}$, and $\mathrm{m} \angle E N Q=(5 z+1)^{\circ}$. Find $\mathrm{m} \angle H E Q$ and $\mathrm{m} \angle E H K$. $\mathrm{m} \angle H E Q=29^{\circ} ; \mathrm{m} \angle E H K=122^{\circ}$


Determine if the conclusion is valid. If not, tell what additional information is needed to make it valid.
14. Given: $\overline{N P} \cong \overline{P Q} \cong \overline{Q M} \cong \overline{M N}$ Conclusion: $M N P Q$ is a square. not valid
Use the diagonals to determine whether a parallelogram with the given vertices is a rectangle, rhombus, or square. Give all the names that apply.
16. $A(-5,7), C(3,6), E(7,-1), G(-1,0)$ rhombus
18. $\mathrm{m} \angle J F R=43^{\circ}$, and $\mathrm{m} \angle J N B=68^{\circ}$. Find $m \angle F B N$. $103^{\circ}$

19. $P V=61.1$, and
$Y S=24.7$.
Find MY. 36.4

20. Find HR. 27 in.


## Answers

9. $X N=Z N=12$, so $\overline{X N} \cong \overline{Z N}$. Thus $\overline{W Y}$ bisects $\overline{X Z} . W N=Y N=15$, so $\overline{W N}$ $\cong \overline{Y N}$. Thus $\overline{X Z}$ bisects $\overline{W Y}$. The diags. of WXYZ bisect each other. By Thm. 6-3-5, $W X Y Z$ is a $\square$.
10. No; 1 pair of opp. sides of the quad. are $\|$. A pair of vert. \&s formed by the diags. are $\cong$. None of the conditions for a $\square$ are met.
11. Possible answer: slope of $\overline{K L}=$ slope of $\overline{S T}=\frac{1}{3}$; slope of $\overline{K T}=$ slope of $\overline{L S}=-\frac{4}{3}$; both pairs of opp. sides have the same slope, so $\overline{K L} \| \overline{S T}$ and $\overline{K T} \| \overline{L S}$; by def., $K L S T$ is a $\square$.
12. Possible answer: $M N P Q$ is a rhombus by def. because its 4 sides are $\cong$. To show that MNPQ is a square, you need to know that MNPQ is also a rect.
